CALCULATION OF THE WAVE PARAMETERS OF TWO-PHASE, ASCENDING, FILM FLOW BY THE METHODS OF STATISTICS

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The use of statistical methods to investigate two-phase film flow in the mode of concurrent ascending flow is demonstrated.

The investigation of the hydrodynamic relationships occurring at the interface of two phases in the case of ascending, two-phase, film flow is necessary for an understanding of the processes of heat and mass transfer as a whole.

As follows from [1, 2], the complicated wave pattern developing at the surface of a liquid film is a consequence of the interaction of various types of waves, each of which is characterized by such parameters as the amplitude, frequency, and phase velocity. The wave parameters of film flow are usually found on the basis of curves of the distribution of a random quantity. To remove elements of subjectivism in such a treatment of experimental data it is necessary to use the mathematical apparatus of the theory of the statistical evaluation of parameters.

The pattern of wave film flow was recorded using the method of local electrical conductivity [1, 2], which makes it possible to record on an oscillogram the instantaneous thickness of a liquid film in the form of a continuous curve.

Under the assumption that the given realization is a steady-state random process, the latter can be described with the help of statistical functions: the mean value of the square of the random process, the distribution density, the correlation function, and the spectral density.

The use of spectral analysis for the treatment of experimental data allows one to perform selection both with respect to frequencies and with respect to amplitudes. In [1, 2], it was shown that the distribution curves of the instantaneous thicknesses of a liquid film constructed on the basis of histograms are close to a normal distribution law. We are confined only to the correlation theory of random functions, in which case Khinchin steadiness (steadiness in the broad sense) is satisfied if the mathematical expectation and dispersion of the random function are constant and its correlation function depends only on the difference in times and not on the absolute values of the times [3, 4]. The construction of histograms for two halves of one realization of the variation of the wave surface of a liquid film showed their agreement, which does not contradict the assumption of the time independence of the probability of the Gaussian distribution. From the assumption that the process is in a stable state it follows that the combined probability density depends only on the difference in times, while the steadiness of the initial series follows from the time independence of the one-dimensional and two-dimensional probability densities. In accordance with the foregoing, we designate as X(t) the steady random process, ergodic with respect to the correlation function, which in this case can be calculated from a single realization x(t) [3, 4].

A correlation transformation is especially effective for eliminating the influence of random interference. We assume that the initial random process contains some periodic components, i.e.,

$$x(t) = s(t) + n(t),$$

where s(t) is some periodic component, and n(t) is the random component. The correlation function of this process will also be a sum of a periodic and a random function, but if n(t)

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does not in turn contain hidden periodic frequencies, then the correlation function of the component n(t) is a decreasing function of the delay $|\tau|$ [5], and for large enough $|\tau|$ the harmonic component stands out in pure form.

The spectral density was calculated numerically through a Fourier transformation of the correlation function, which is of independent interest, as shown above.

Using the elements of spectral analysis, we treated the experimental data on the wave surface of a liquid film in the mode of ascending concurrent flow for systems of water and air, an aqueous glycerin solution and air, and an aqueous caprolactam solution and air within the limits of trickling densities from 0.2 to 7 cm^2/sec and gas velocities from 11 to 60 m/sec.

The normalized correlation function $\overline{R}(\tau) = R(\tau)/R(0)$ of one realization for the system of an aqueous glycerin solution and air at V = 34.1 m/sec and q = 0.4 cm^2/sec is shown in Fig. 1. The behavior of $\overline{R}(\tau)$ displays some periodicity and the absence of damping, which, as was discussed above, is characteristic of the periodic component in the given process being analyzed. The numerical values of the parameters of the wave flow can be obtained from the form of the spectral density $S(\omega)$; in particular, from the location of the peaks in $S(\omega)$ one can determine the values of the frequencies of the periodic components in the initial process. Since the process being analyzed is assigned in a finite interval [−L, L], the results of the correlation transformation only give an estimate of the spectral density.

A smoothed estimate of the normalized spectral density is presented in Fig. 2. One actually observes the presence of a certain number of peaks in the frequency dependence of the spectral density. One must keep in mind, however, that in the calculation of the spectral density the smoothing was done using a Tukey window [3, 4], and the result of the action of any spectral window is characterized by the fact that narrowing the width of the spectral window leads to a decrease in the shift, i.e., the more clearly the sections in the spectral density with frequencies close to ω_0 (one of the periodic frequencies) are isolated, the larger the dispersion proves to be. All this leads to the appearance of spurious peaks in the dependence $S(\omega)$. The reliability of one or another peaks can be established using the confidence intervals (Fig. 2). With allowance for this, we can consider f_1 = 24 Hz, f_2 = 128 Hz, and f_3 = 350 Hz as the most probable frequencies connected with periodic components in the initial wave process. In this way three main types of waves are observed in the process, which can be treated as perturbation waves, large waves, and ripple waves, which was also noted in [1, 2].

Thus, the introduction of such parameters as the amplitude, wavelength, and frequency to describe the wave structure of a liquid film is quite proper, since they correspond to actually existing periodic components in the variation of the wave surface of a liquid film in the mode of ascending concurrent flow.

The presence of three main types of waves in the wave process during two-phase film flow in a pipe was shown earlier [1] on the basis of the distribution law only. The latter fact is also confirmed by the form of the spectral density (Fig. 2).

In connection with the fact that the hydraulic resistance and the mass exchange in two-phase film flow [6] essentially depend on the wave parameters, it is interesting to determine them on the basis of the bringing out of hidden periodicity, using for this the well-known canonical form of polyharmonic expansion [7] for a wave process x(t), assigned in the finite interval [−L, L]:

$$y(t) = A_0 + \sum_{j=1}^{m} (A_j \cos \omega_j t + B_j \sin \omega_j t). \tag{1}$$

If N ($N \geq 3m + 1$) values $y_1 = y(1)$, $y_2 = y(2)$, ... $y_N = y(N)$ are known from experiment, then $\cos \omega_1$, $\cos \omega_2$, ..., $\cos \omega_m$ are determined as the roots of the algebraic equation

$$\cos m\omega - \alpha_1 \cos (m-1)\omega - \ldots - \alpha_{m-1} \cos \omega - \frac{1}{2} \alpha_m = 0,$$

whose coefficients α_k must satisfy the system of linear equations

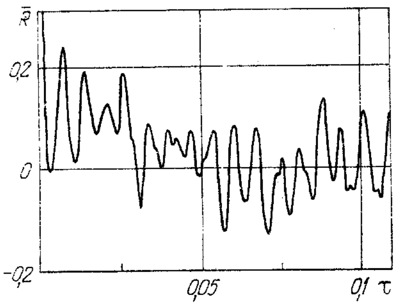

Fig. 1

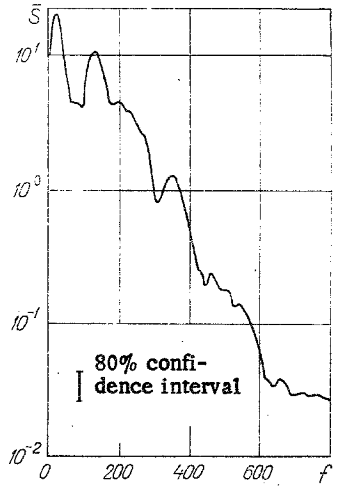

Fig. 2

Fig. 1. Dependence of normalized correlation function $\overline{R}$ on delay τ, sec.

Fig. 2. Dependence of normalized spectral density $\overline{S}$ on frequency f, Hz.

$$\varepsilon_i = \sum_{k=1}^{m-1} (y_{i+k} + y_{2m+i-k})\,\alpha_k + y_{m+i}\alpha_m - y_i - y_{2m+i} = 0$$

$$(i = 1,\ 2,\ \ldots,\ N-2m).$$

To find the m coefficients α_k by the method of least squares one must solve the system of m linear equations

$$\frac{\partial}{\partial \alpha_k} \sum_{i=1}^{N-2m} \varepsilon_i^2 = 0.$$

A_j and B_j are determined from the equations

$$A_j = \frac{2}{m} \sum_{k=1}^{m} y_k \cos j\,\frac{2\pi k}{m}; \quad B_j = \frac{2}{m} \sum_{k=1}^{m} y_k \sin j\,\frac{2\pi k}{m},$$

where $0 \leqslant j \leqslant m/2$.

All the parameters A_j, B_j, and ω_j, which number 3m + 1 in the general case, can be determined from this algorithm using a computer. Since the number of periodic components is now known in advance, their choice is based on the results of [1] and of a spectral analysis on the presence of three main types of waves. Consequently, we set m = 3 in Eq. (1). Since the algorithm described is based on the method of least squares, it provides the isolation of the most probable average frequencies, three in the given case.

As a result of calculations for realizations with different phase loads, we isolated three frequencies, each of which always fell in the appropriate range of frequencies connected with perturbation waves, large waves, and ripple waves.

As an example in the calculation of the spectral density we found $f_1' = 11$ Hz, $f_2' = 102$ Hz, and $f_3' = 308$ Hz by the method of least squares for the realization which was used in the present work.

In conclusion, it is interesting to compare the frequencies connected with the three main types of waves and obtained by the two methods described above with the frequencies found earlier [1, 2] on the basis only of the distribution curves, which, for phase loads analogous to those of the example considered above, equal $f_p = 19.3$ Hz for perturbation waves, $f_l = 104$ Hz for large waves, and $f_r = 312$ Hz for ripple waves. Rather good agreement of the frequencies obtained by the three different methods is seen.

The calculations performed show that statistical methods are a reliable means for determining the wave parameters of two-phase, ascending, film flow.

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LONGITUDINAL DIFFUSION OF AN IMPURITY IN A PIPE WITH PERMEABLE WALLS

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The problem of the spread of an impurity in a stream with fluid injection through the pipe walls is analyzed.

The theory of convective longitudinal diffusion of a passive impurity in laminar and turbulent streams during fluid flow in pipes with impermeable walls was developed by Taylor [1, 2]. In [3] the solution of this problem was constructed by the method of successive approximations; in this way it was shown that Taylor's model is asymptotically valid as $t \to \infty$. In the present report the solution of the diffusion equation is obtained for long times in the presence of fluid injection through permeable pipe walls.

The distribution of a passive impurity in laminar and turbulent modes of flow in flat and round pipes without allowance for diffusion in the longitudinal direction is described by the equation

$$\frac{\partial c}{\partial t}+u_x\frac{\partial c}{\partial x}+u_r\frac{\partial c}{\partial r}=\frac{D}{r^\alpha}\frac{\partial}{\partial r}\left(r^\alpha\gamma(r)\frac{\partial c}{\partial r}\right). \tag{1}$$

For hydrodynamically stabilized flow in the presence of injection the longitudinal and radial velocity components can be expressed through one function $F(\eta;\ \mathrm{Re}_V)$ [4, 5]:

$$u_x=\left(U_0+\frac{2^\alpha Vx}{r_0}\right)\frac{F'(\eta)}{(2\eta)^\alpha},\quad u_r=-V\frac{F(\eta)}{\eta^\alpha}. \tag{2}$$

With allowance for (2), Eq. (1) takes the dimensionless form

$$\frac{\partial c}{\partial\tau}+\frac{\mathrm{Pe}}{2}\frac{\partial c}{\partial X}+\frac{\mathrm{Pe}}{2}\left(\frac{F'(\eta)}{(2\eta)^\alpha}-1\right)\frac{\partial c}{\partial X}-\frac{P}{2}\frac{F(\eta)}{\eta^\alpha}\frac{\partial c}{\partial\eta}=\frac{1}{\eta^\alpha}\frac{\partial}{\partial\eta}\left(\eta^\alpha\gamma(\eta)\frac{\partial c}{\partial\eta}\right). \tag{3}$$

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